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How to spread Information in Social Networks that contain Adversarial Users

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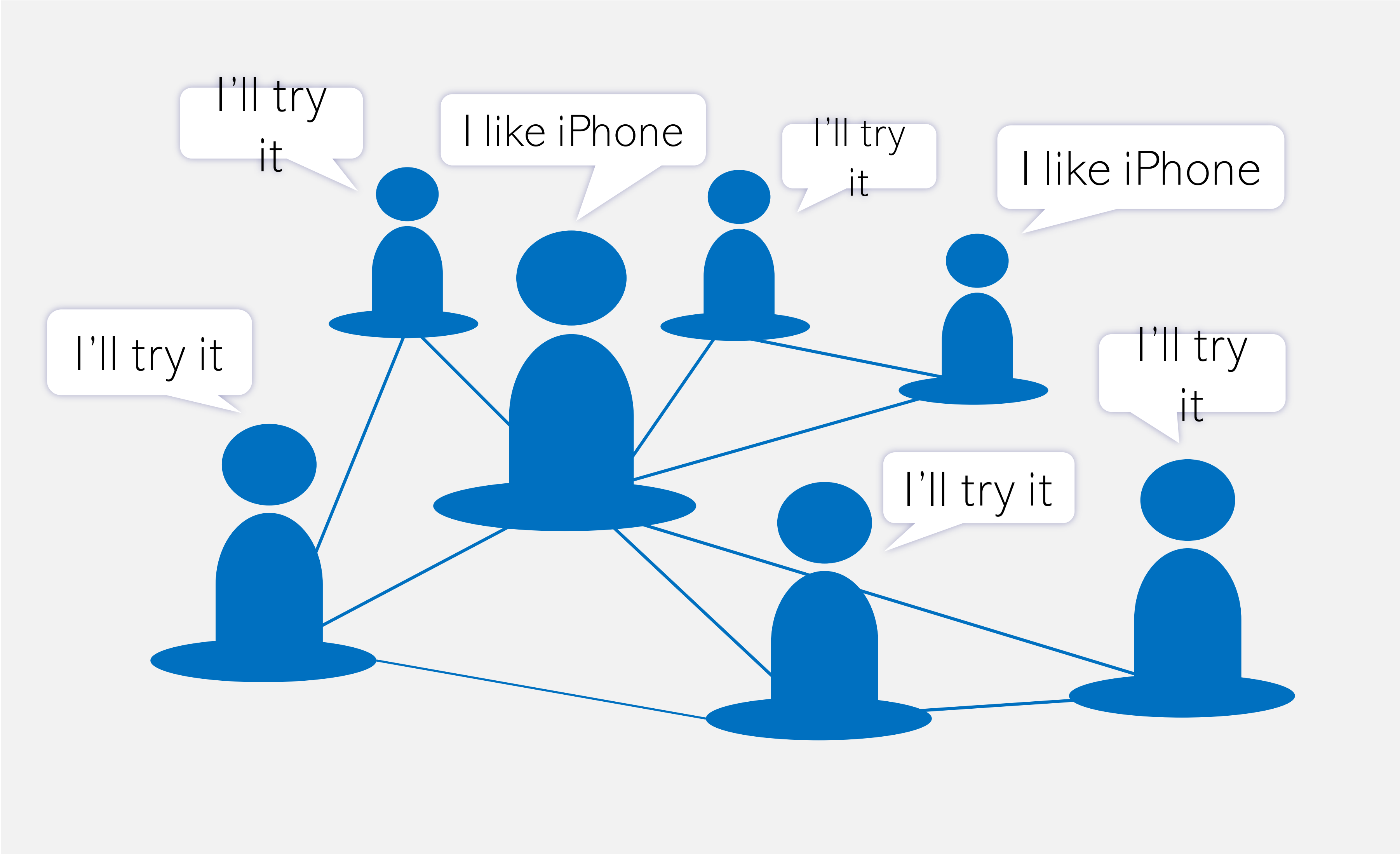
Outline

- Overview of Influence Maximization
- CIM Problem
- Greedy Algorithm
- MultiGreedy Algorithm
- Experimental Results

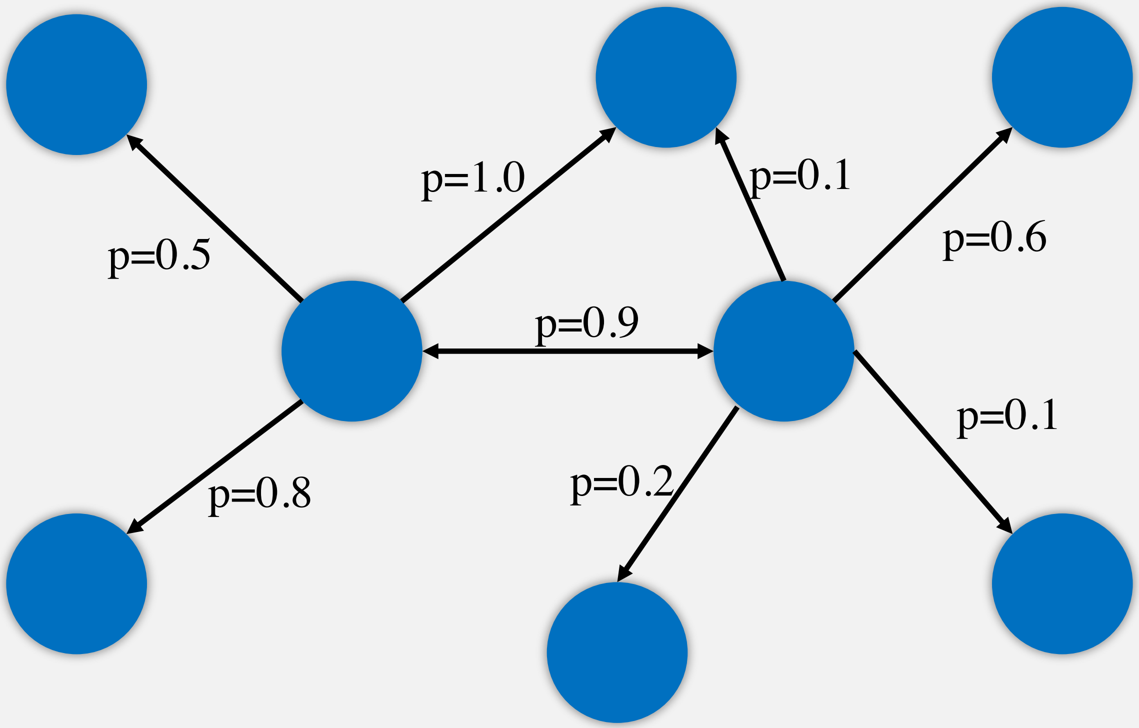
OVERVIEW OF INFLUENCE MAXIMIZATION

Social Networks



Influence Maximization Problem

- Find a set of highly influential users *(seed)* in the network
- Posed as an optimization problem by Kempe et. Al.



Applications

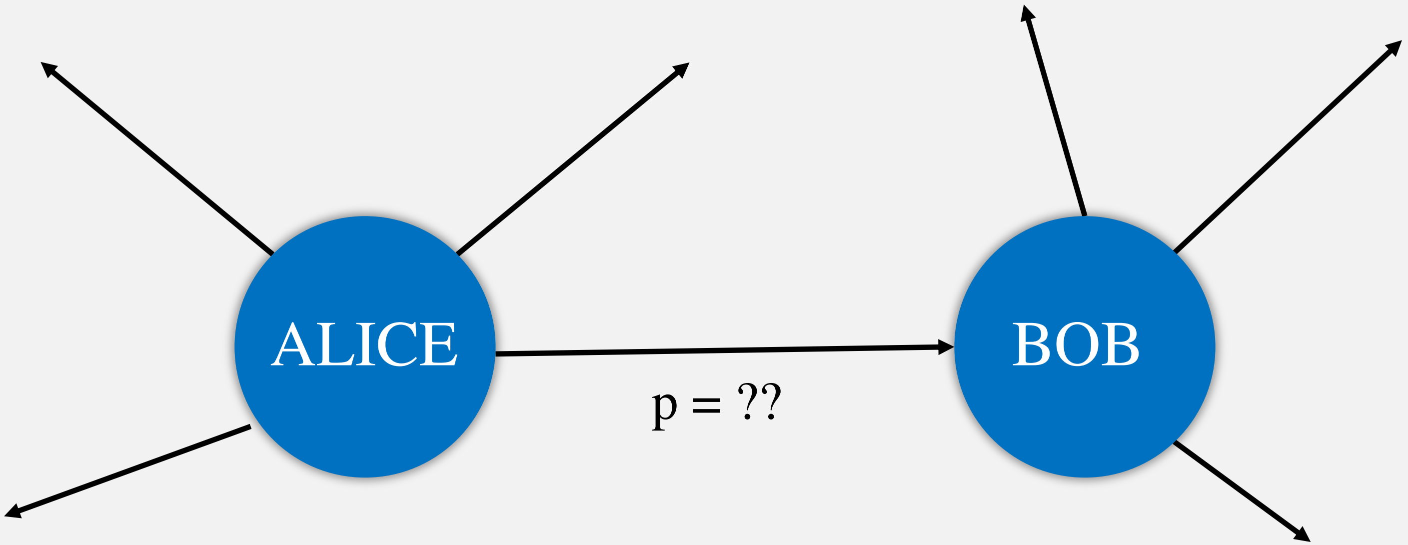
- Political Campaigning – How can I get people to vote for me?

- Viral Marketing – Who do I ask to advertise a product?

Information Diffusion

1. An “idea” originates from a user or a set of users in the network. These users are called the ***“Seed”*** users.
2. Users connected to the ***Seed*** are exposed to the idea.
3. The exposed users, if they choose to, further propagate the idea to users connected to them.

Diffusion – A Probabilistic Process



- Alice posts about something !
- What’s the chance that Alice influences Bob?
- What’s the chance Alice influences Bob’s friends?

Diffusion Models

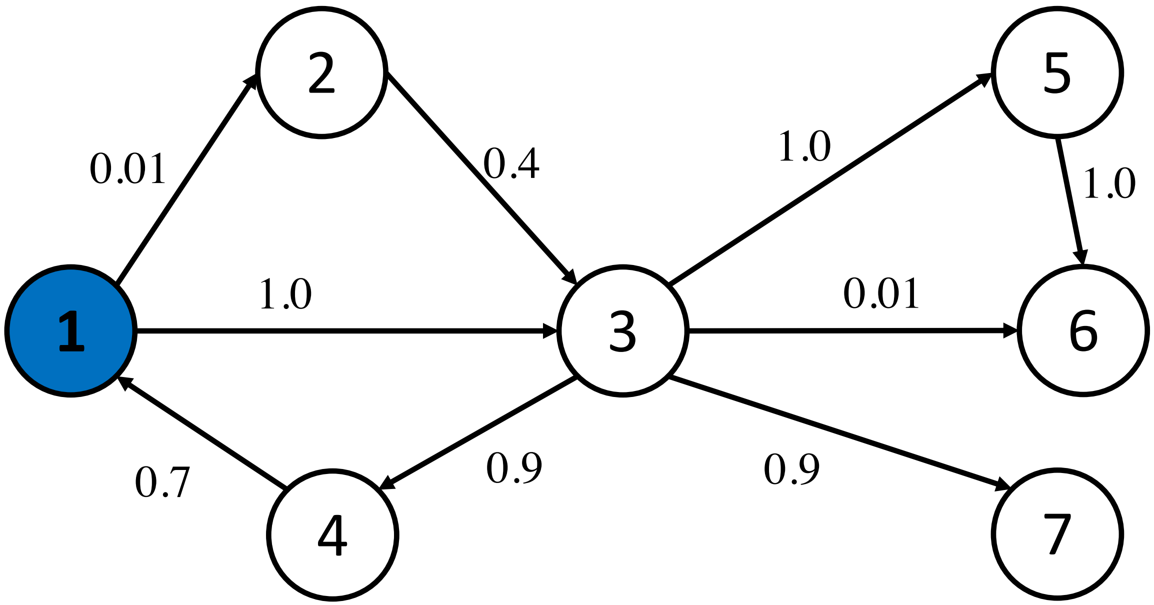
- Models of *Diffusion* – Characterizes the spread of information from one user to the next
- Models mirror the diffusion in real world social networks.
- 2 popular models:
  - Independent Cascade (IC) Model
  - Linear Threshold (LT) Model

Independent Cascade Model

- The network is modelled as a graph $G = (V, E)$.
- Every edge (u, v) has an associated probability - $p(uv)$.
- u has exactly one chance to convince v to adopt the idea.

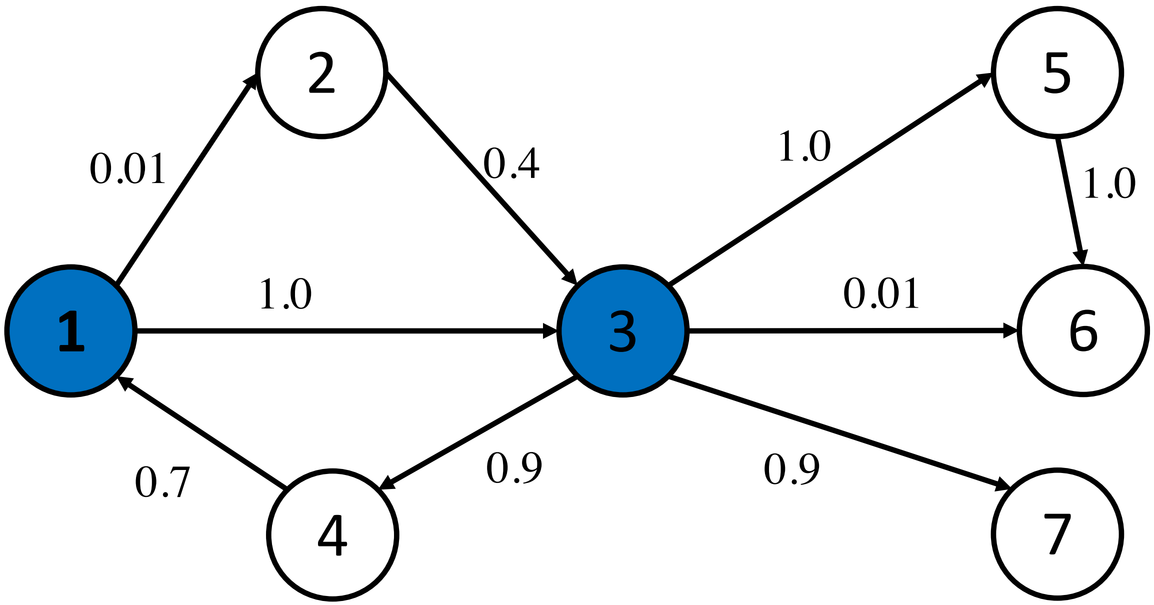
  u succeeds with probability $p(uv)$.
- If successful, v is considered to be ***"activated"***.

The Independent Cascade Model – an example



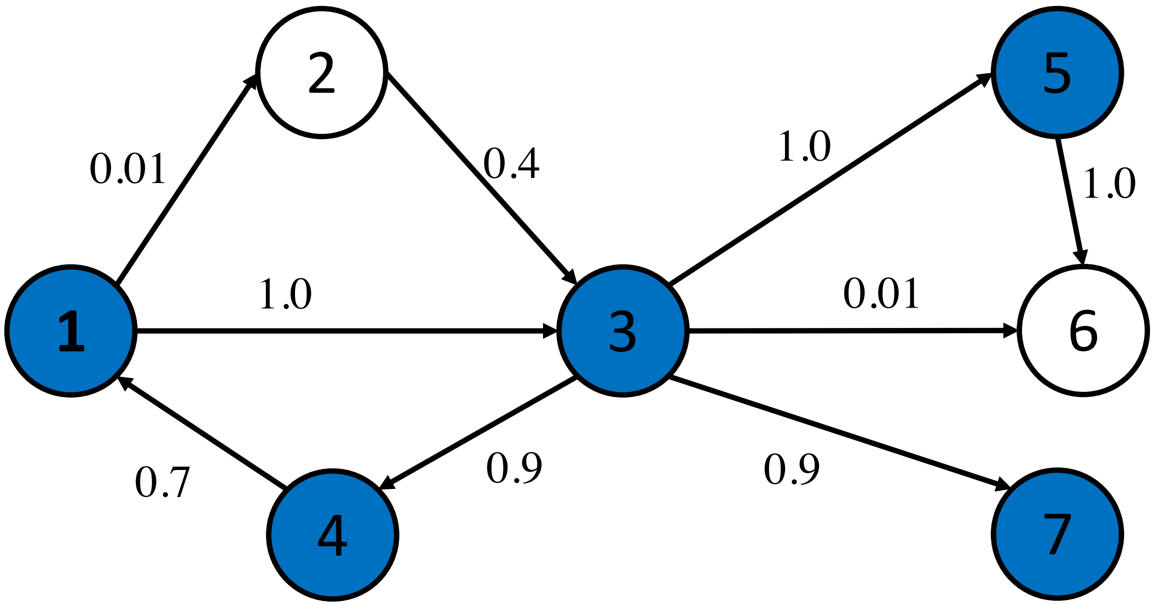
$t = 0$

The Independent Cascade Model – an example



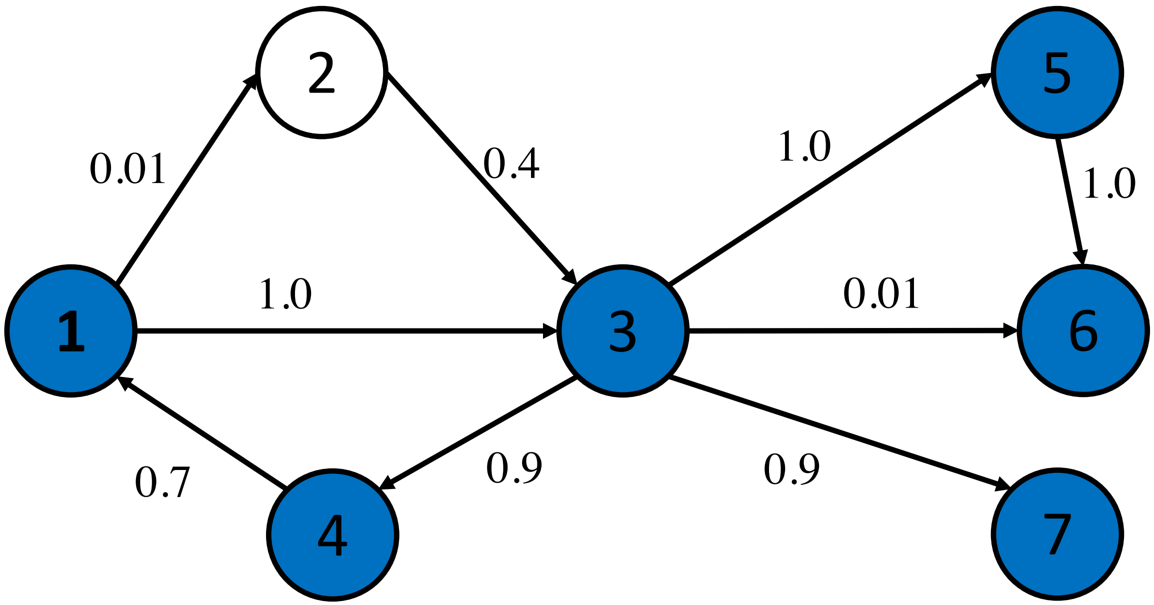
t = 1

The Independent Cascade Model – an example



t = 2

The Independent Cascade Model – an example

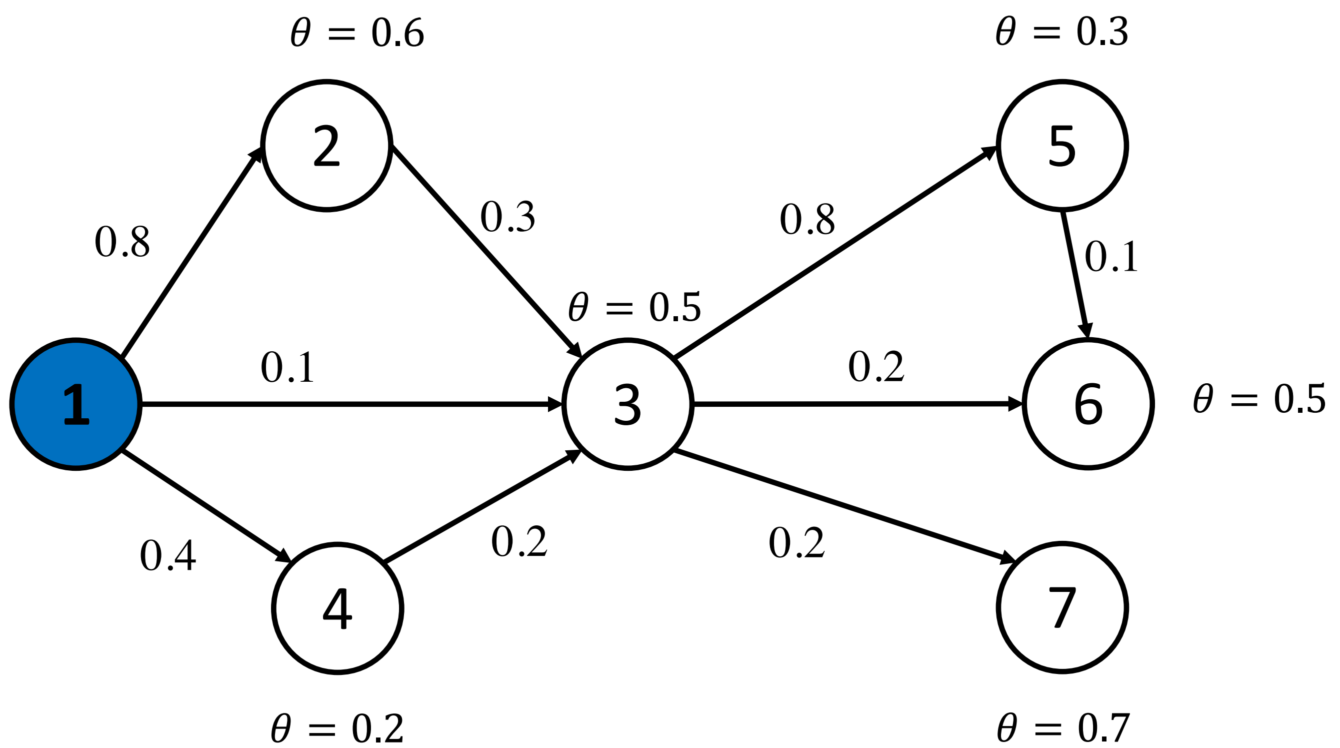


t = 3

Linear Threshold Model

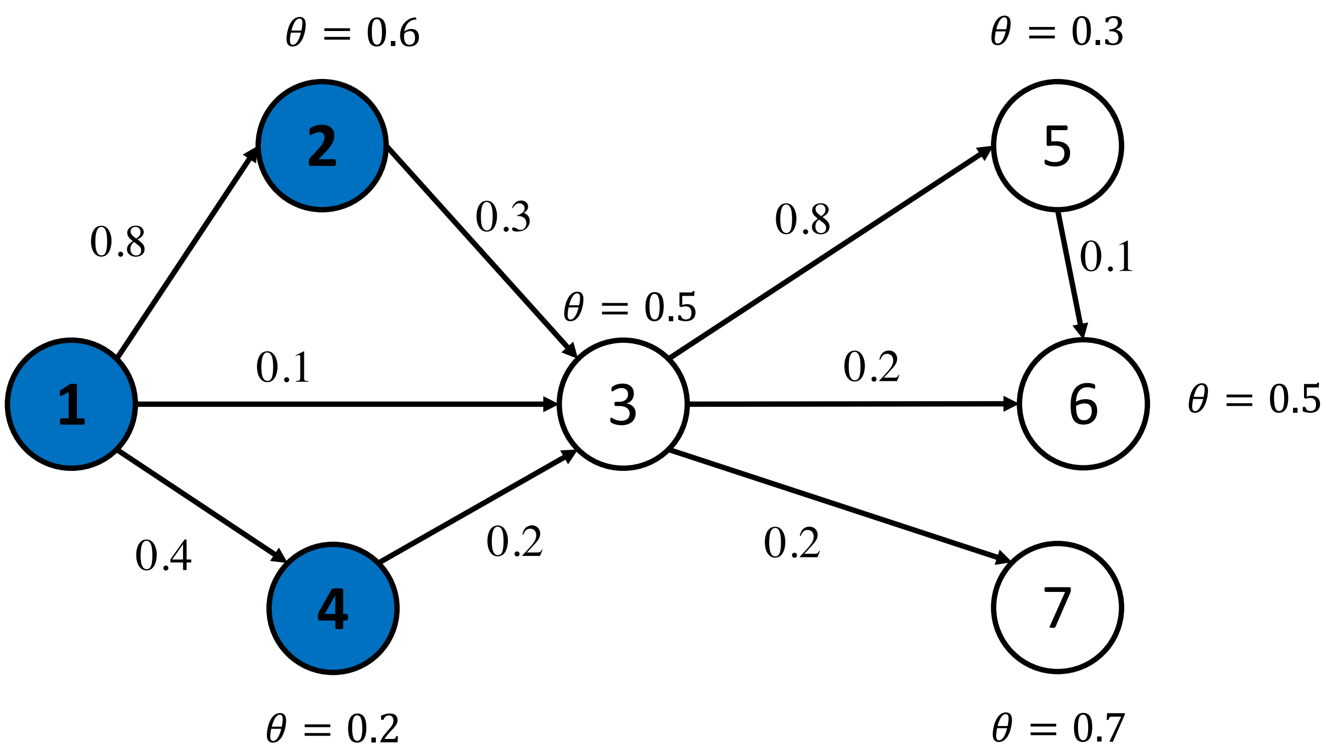
- The network is modelled as a graph $G = (V, E)$.
- Every edge (u, v) has an associated weight - w(u, v).
- Each node v is assigned a random *threshold* $\theta_v \in [0,1]$.
- v is considered to be ***"activated"*** if sufficient neighbors of v are activated:
  - $\boldsymbol{\sum_{Active\ u} w(u, v) \geq \theta_v}$

The Linear Threshold Model – an example



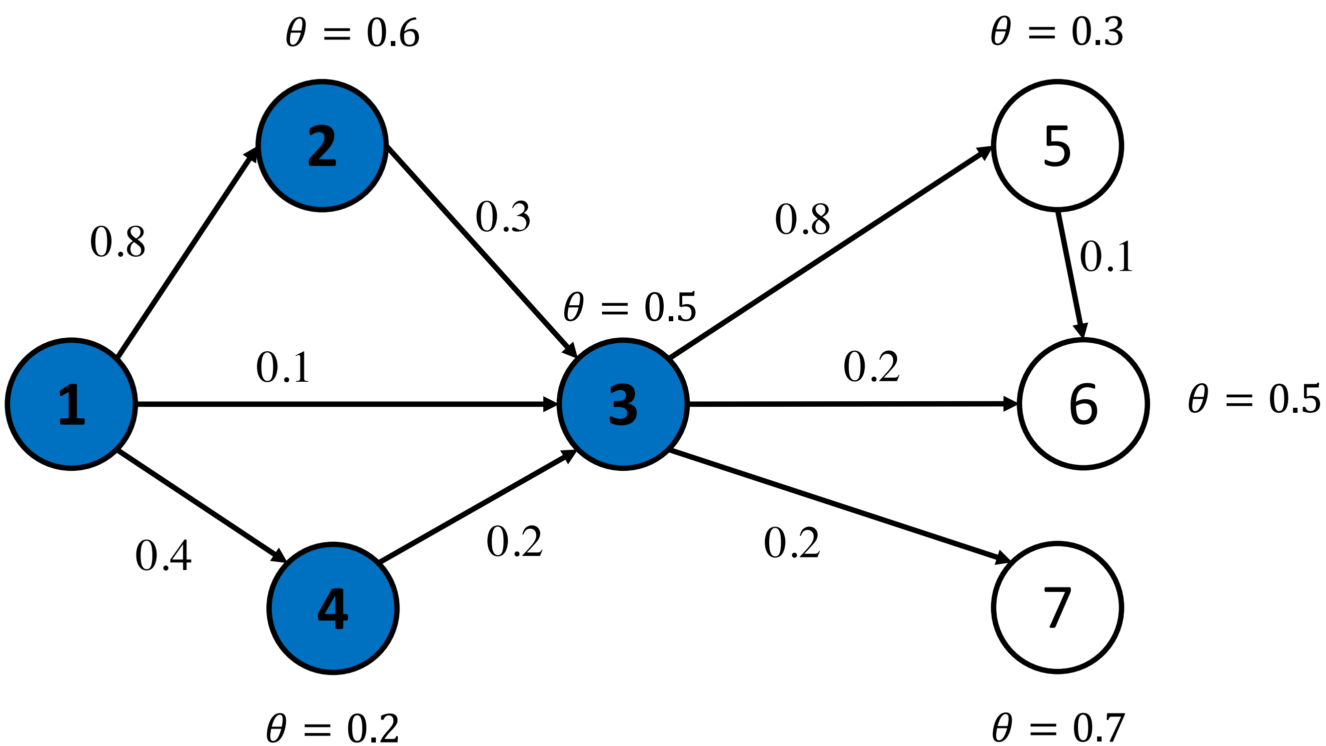
t = 0

The Linear Threshold Model – an example



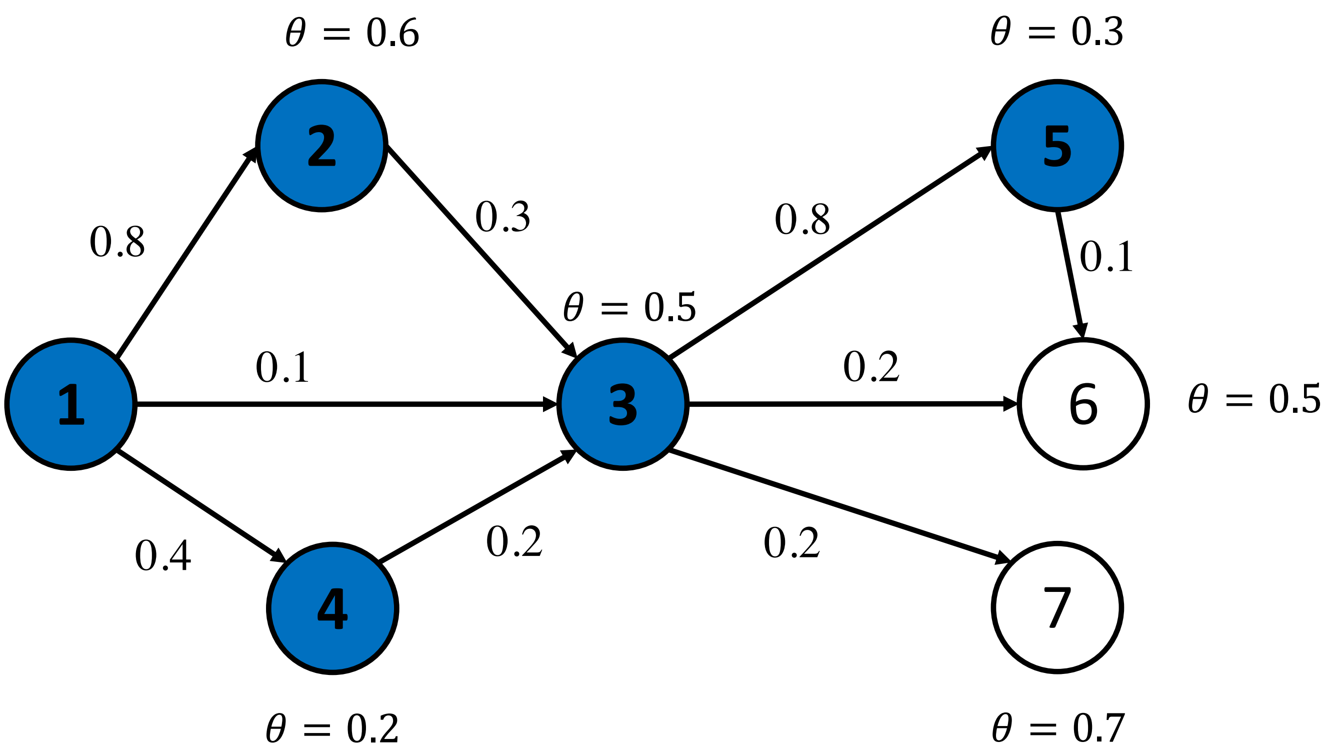
t = 1

The Linear Threshold Model – an example



t = 2

The Linear Threshold Model – an example



t = 3

Influence Function

- $\sigma(S)$ - *Expected number of users influenced by a set of users S*
- *Under the IC Model,* $\sigma(S)$ is *monotone, submodular*
- *Submodular:*

$$\forall X \subset Y \subseteq V \ and \ a \notin Y:$$

$$\sigma(X \cup \{a\}) - \sigma(X) \geq \sigma(Y \cup \{a\}) - \sigma(Y)$$

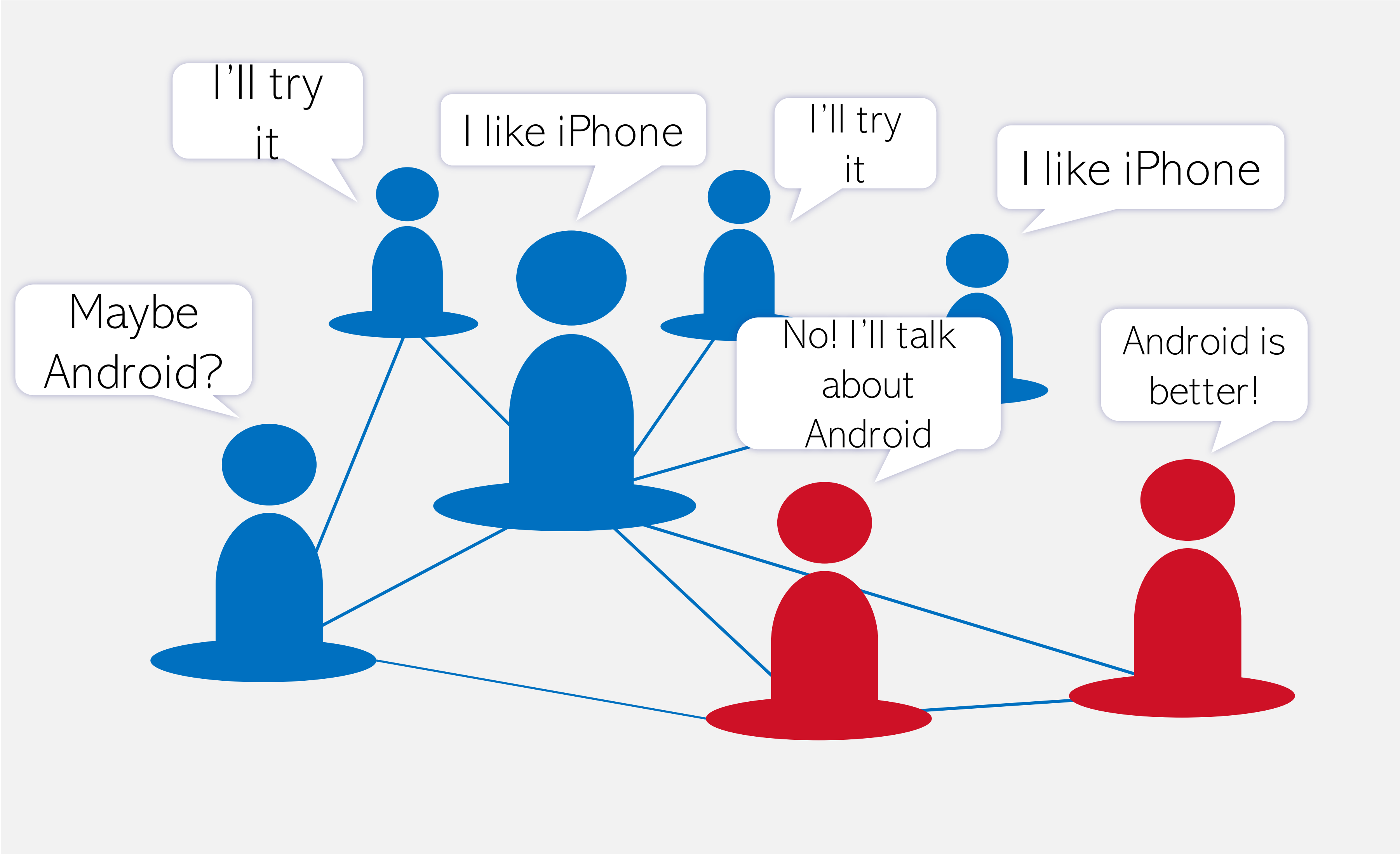
Influence Maximization

- **Input**: $G=(V,E)$, a budget k
- $\sigma(S)$ - *Expected number of users influenced by a set of users S*
- ***Objective***:

$$Find\ S\ of\ size\ k\ that\ maximizes\ \sigma(S)$$

- An NP-Hard problem
- *Greedy algorithm gives a 0.63-approximate solution*

What if there are adversaries?



What if there are adversaries?

- Political Campaigning – Can rally opposing candidate supporters
- Marketing: Advertisements for products such as alcohol, tobacco must not be shown to children
- Can cause a negative reaction to the information being spread
- How to approach this problem?

CIM PROBLEM

Constrained Influence Maximization(CIM)

- Label users as “Targets” or ”Non-Targets”
- Given: *G=(V,E, L),* budget *k,* threshold θ
- $\sigma_T(S)$ *- Expected number of “Target” users influenced by S*
- $\sigma_N(S)$ *- Expected number of “Non-Target” users influenced by S*
- $\sigma^\theta(S) = \begin{cases} \sigma_T(S), \; \sigma_N(S) \leq \theta \\ 0, \; otherwise \end{cases}$
- Objective:

$$Find\ S\ of\ size\ k\ that\ maximizes\ \sigma^\theta(S)$$

IM vs. CIM

Influence Maximization Problem

$$Maximize\ \sigma(S)$$
$$s.t.\ |S| \leq k$$

- $\sigma(S)$ is a monotone, submodular function.
- Greedy Algorithm gives a 0.63-approximate solution.

IM vs. CIM

Constrained Influence Maximization Problem

$$Maximize\ \sigma_T(S)$$
$$s.t.\ \sigma_N(S) \leq \theta$$
$$|S| \leq k$$

- $\sigma_T(S), \sigma_N(S)$ are monotone, submodular functions.
- Maximize under a submodular constraint and a cardinality constraint !

IM vs. CIM

IM Problem	CIM Problem
• NP-Hard	• NP-Hard
• Submodular Maximization	• Submodular Maximization
• Cardinality Constraint	• Submodular Constraint
• Greedy algorithm gives a 0.63-approximate solution.	• Cardinality Constraint

Theoretical Challenges of CIM

- **Theorem**: For every 0$\leq c \leq 1$, if there is a polynomial time c-approximation algorithm for the CIM problem under the IC model, then every problem in NP can be solved in $O\left(n^{(\log n)^k}\right)$ time, for some k≥ 1.

Obtaining a constant factor approximation algorithm is quasi NP-hard!

NATURAL GREEDY ALGORITHM

Natural Greedy Algorithm

Start with an empty Set

↓

Find the best vertex v that when added, the set influences at most θ Non Targets

↓

Repeat until a set of size k is obtained

Theoretical Analysis of Greedy

Theorem:

$$\sigma_T^{\theta}(S) \geq 0.63\ OPT - Additive\ Loss$$

The Greedy solution has an approximation guarantee that depends on an additive error!

Runtime: $O(k \times |V| \times Time\ taken\ to\ compute\ \sigma)$

Theoretical Analysis of Greedy

Proof Idea:

Let S^* be the set that has the optimum value $OPT_{k,\theta}$.

$gain_{S_i}(\{e\},\ \theta)$– The gain achieved by adding $\{e\}$ to S_i such that at most θ Non-Targets are influenced.

$$OPT_{k,\theta} \leq \sigma^{2\theta}(S^* \cup S_i)$$

$$OPT_{k,\theta} \leq \sigma^{\theta}(S_i) + \sum_{e \in S^*} gain_{S_i}(\{e\}, 2\,\theta)$$

$$OPT_{k,\theta} \leq \sigma^{\theta}(S_i) + k \times \sigma^{2\theta}(S'_{i+1}) - k \times \sigma^{\theta}(S_i)$$

Theoretical Analysis of Greedy

$BG(S_i, \theta)$ – The maximum gain achieved by adding an element to S_i such that at most θ Non-Targets are influenced

$$OPT_{k,\theta} - \sigma^{\theta}(S_{i+1}) \leq \left(1 - \frac{1}{k}\right)(OPT_{k,\theta} - \sigma^{\theta}(S_i)) + BG(S_i, 2\theta) - BG(S_i, 2\theta)$$

$$\sigma_T^{\theta}(S_k) \geq 0.63\, OPT_{k,\theta} - \left(\sum_{i=0}^{k-1} BG(S_i, 2\theta) - \sigma^{\theta}(S_k)\right)$$

Theoretical Analysis of Greedy

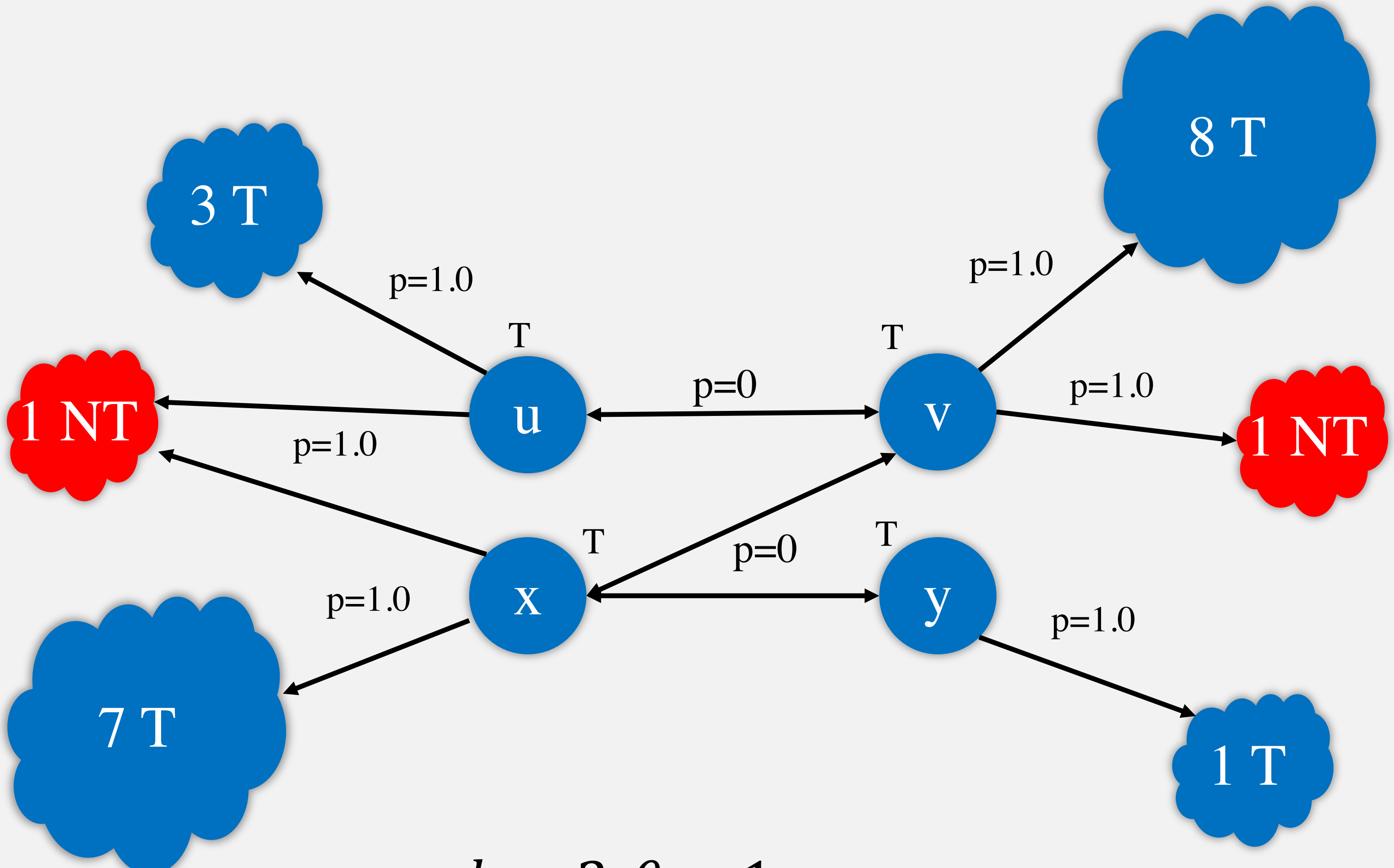
Additive Loss:

$$\sum_{i=0}^{k-1} BESTG(S_i, 2\theta) - \sigma_T^{\theta}(S_k)$$

Approximately difference of targets influenced between by the greedy solution with threshold θ and threshold 2θ

Runtime: $O(k \times |V| \times Time\ taken\ to\ compute\ \sigma)$

Can we improve on Greedy?

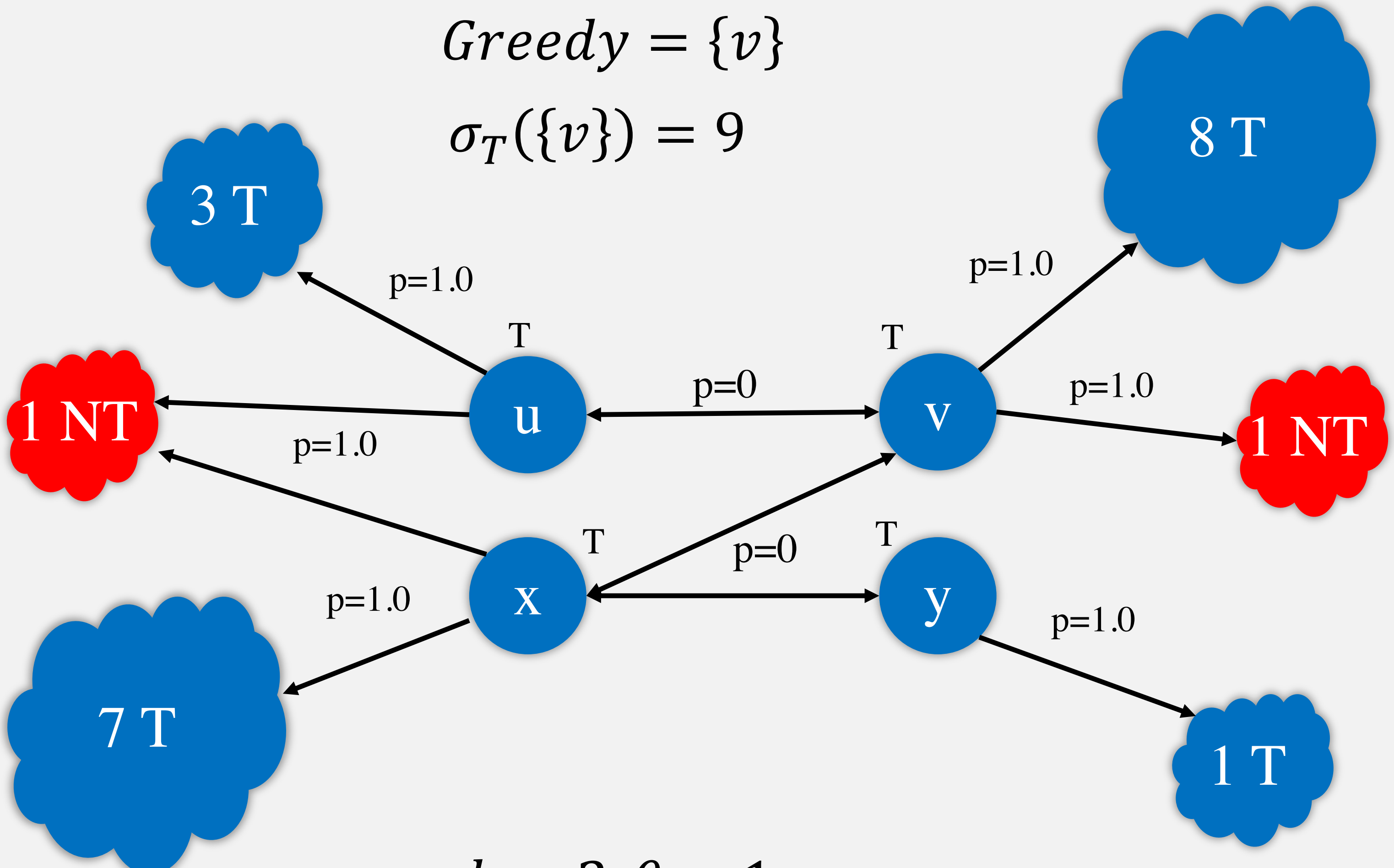


$$k = 2, \theta = 1$$

Can we improve on Greedy?

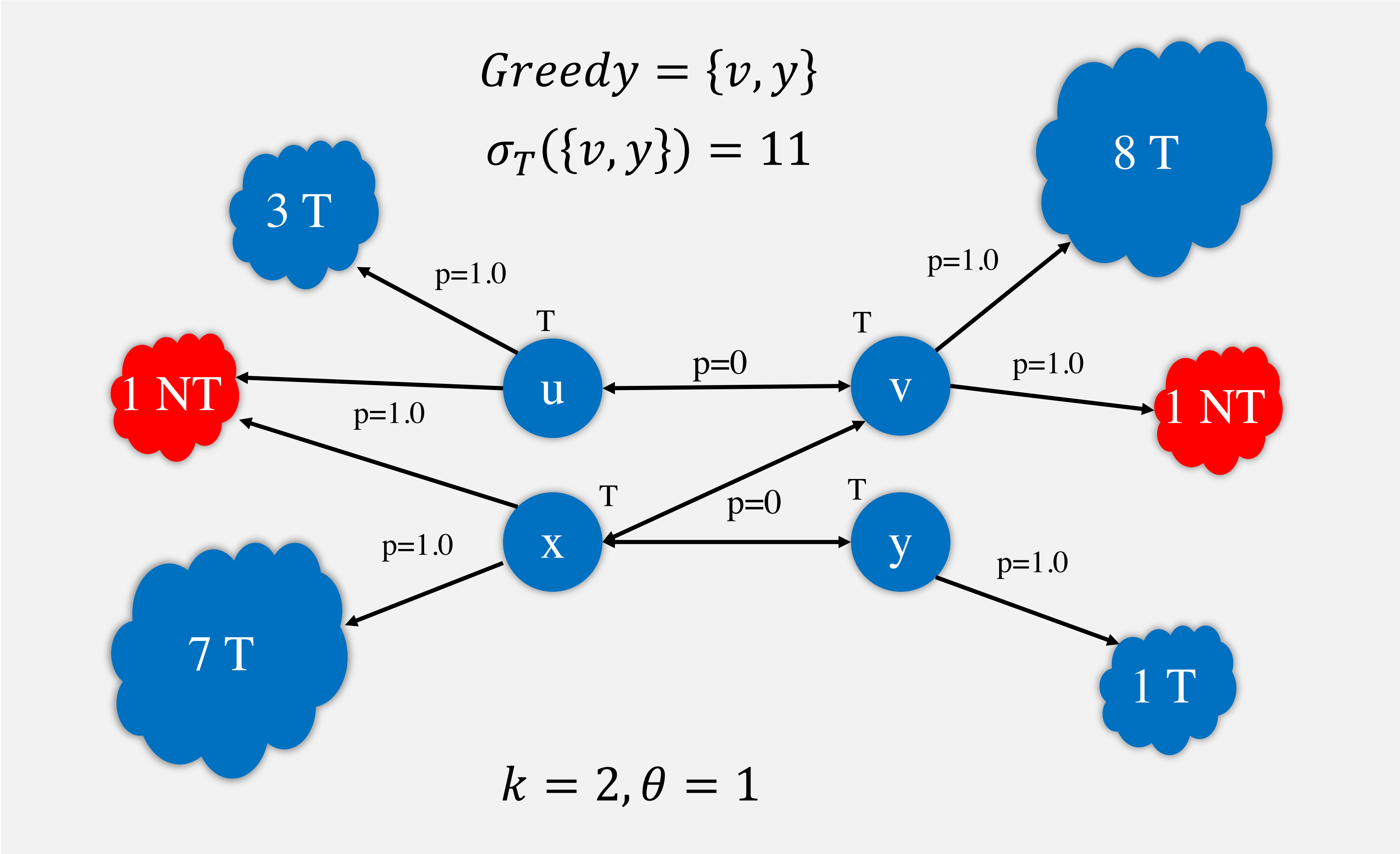
$$Greedy = \{v\}$$

$$\sigma_T(\{v\}) = 9$$

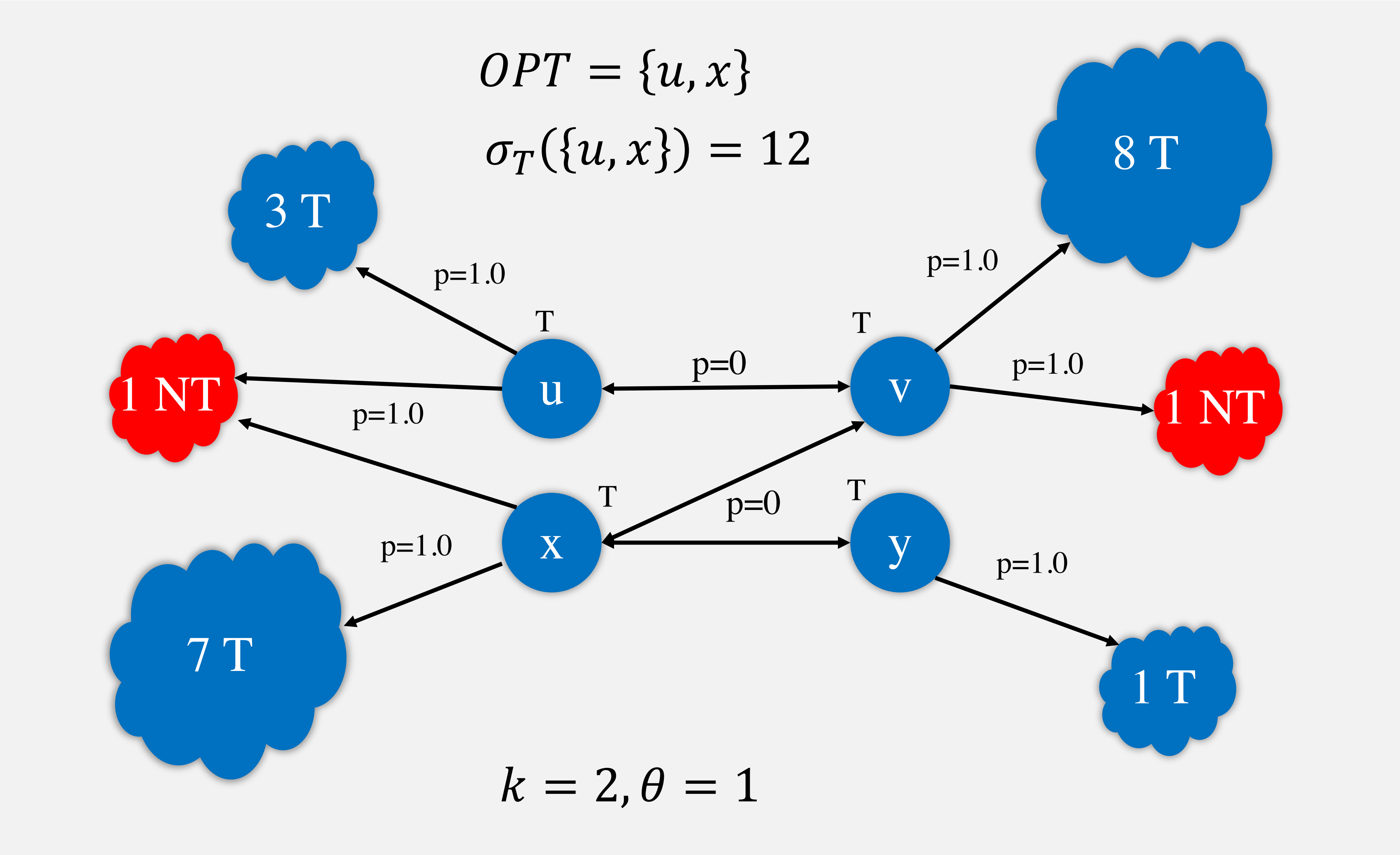


$$k = 2, \theta = 1$$

Can we improve on Greedy?



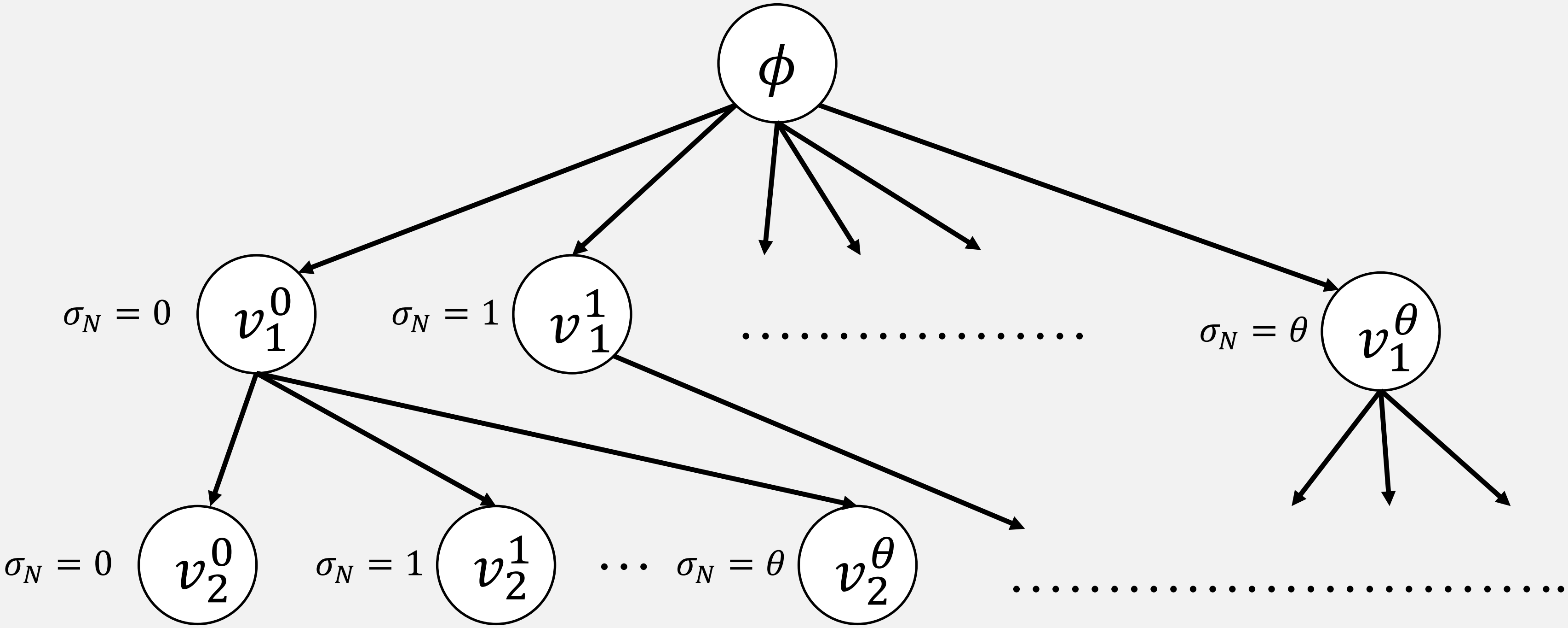
Can we improve on Greedy?



MULTIGREEDY ALGORITHM

MultiGreedy Algorithm

- Keeps track of multiple seed sets



Proceed till depth k and return the best path from root to leaf

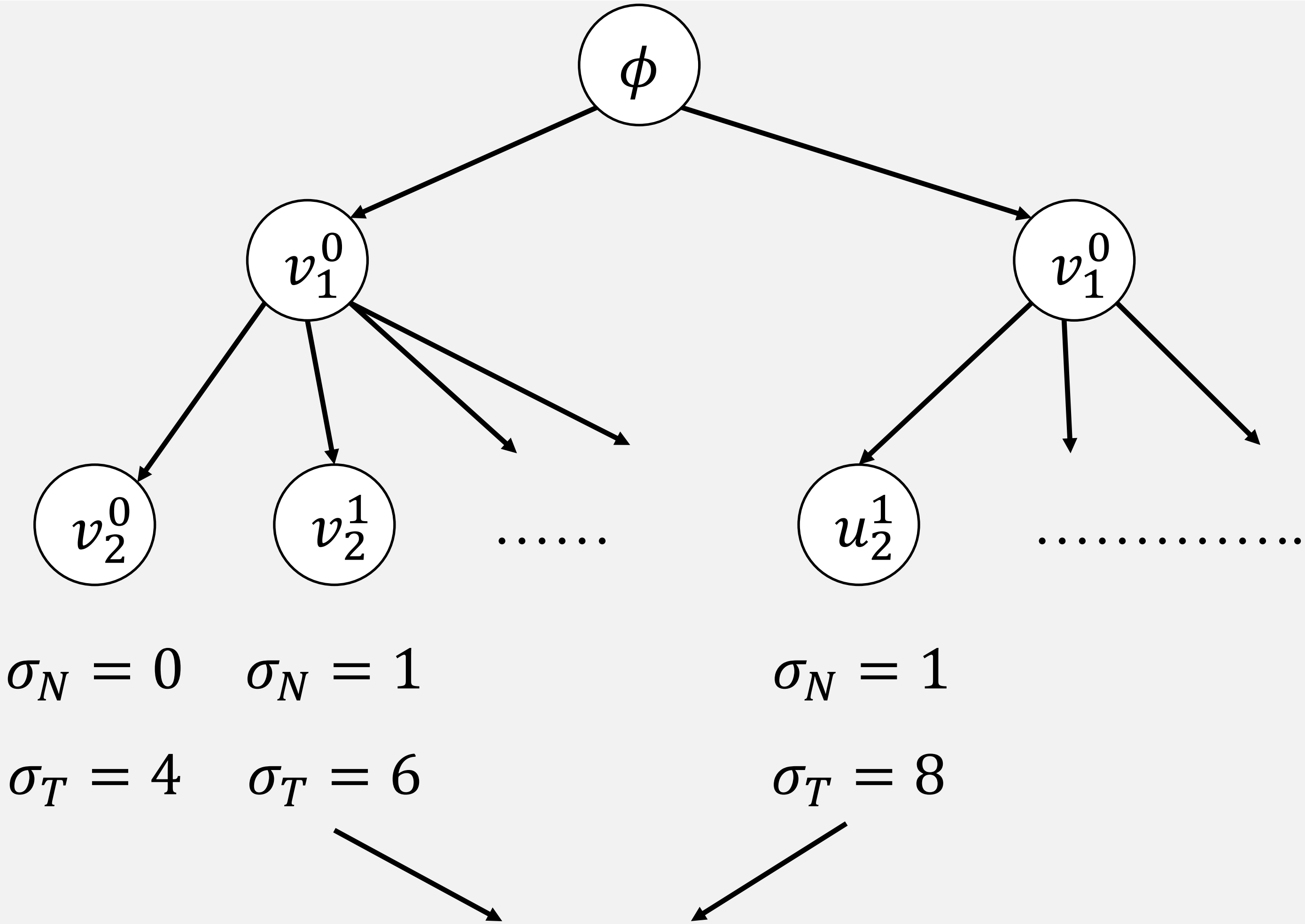
MultiGreedy Algorithm

- The greedy solution will be in one of the branches
- **Theorem:**

$$\sigma_T^\theta(MultiGreedy) \geq \sigma_T^\theta(Greedy)$$

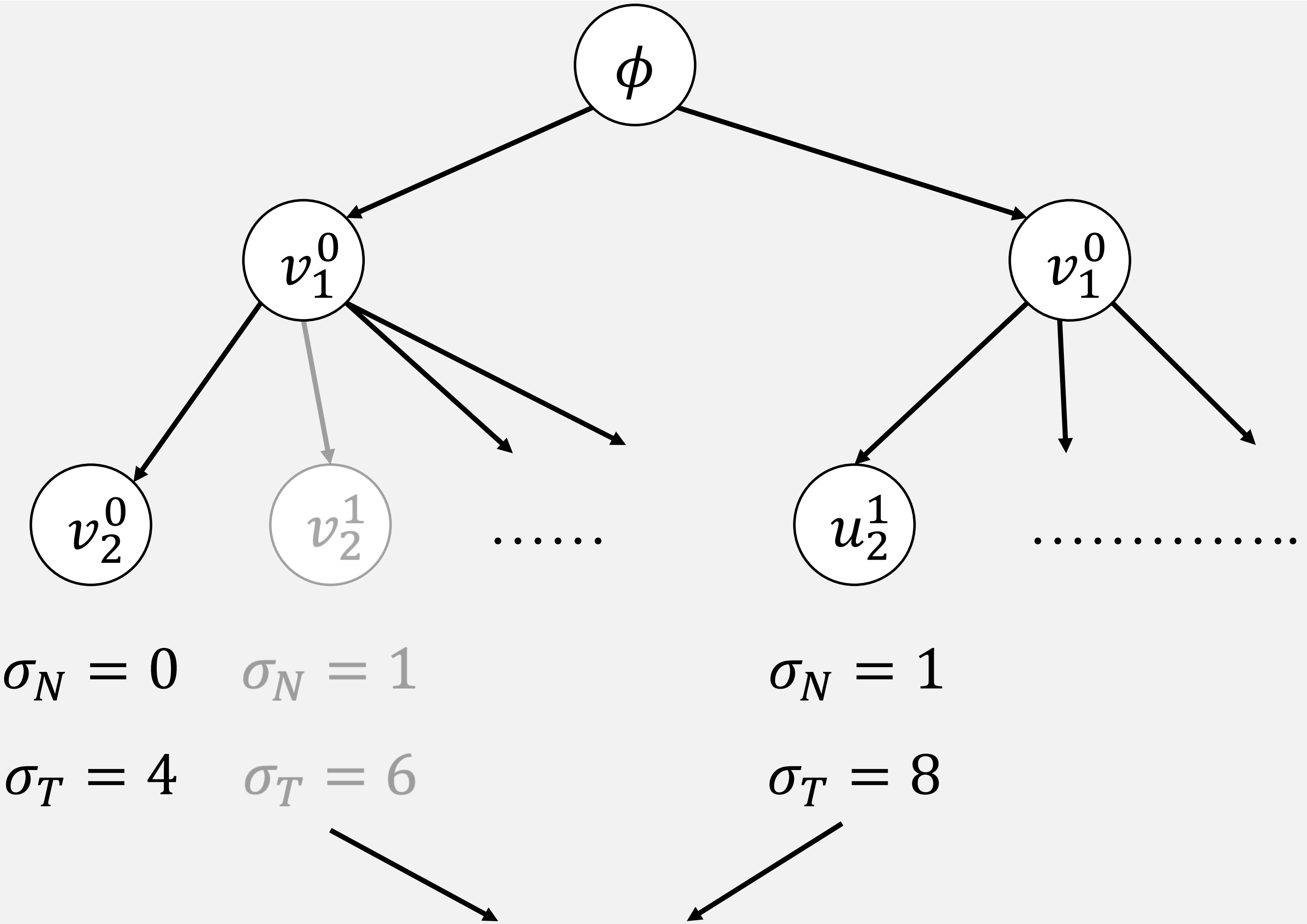
- Runtime: At least $\mathrm{O}(\theta^k)$
- Computationally infeasible!
- Let's prune the tree!

Efficient MultiGreedy with IMTree



Both hit 1 Non Target, but u_2^1 hits 8 Targets !

Efficient MultiGreedy with IMTree



Let's prune the branch that contains v_2^1

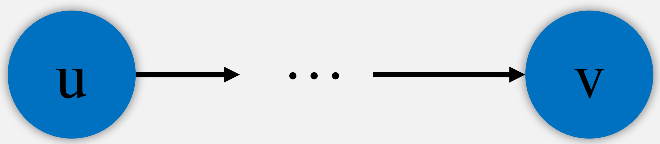
ESTIMATING INFLUENCE FUNCTION $\sigma(S)$

Estimating Influence Function

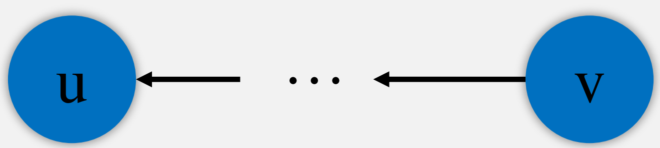
- Exact computation of $\sigma_T(S), \sigma_N(S)$ is #P-Hard
- Several techniques exists: Monte Carlo Simulations, Forward Influence Sketching, Reverse Influence Sketching(RIS)
- We've used RIS based estimation.
- Our algorithms can be adapted to different methodologies of estimating the influence function

Reverse Influence Sampling

- Let $g \sim G$ be a graph sampled from the random graph distribution
- $P[\, u\ influencing\ v] = P[\exists\ path\ from\ u\ to\ v\ in\ g]$



- Look at transpose g^T!
- $P[\, u\ influencing\ v] = P[\exists\ path\ from\ v\ to\ u\ in\ g^T]$



Random Reverse Reachable Set

- Randomly Select a vertex u.
- Generate a set R by performing a Random Reverse BFS starting from u
- $\sigma(S) = n \times P[S \cap R \neq \phi]$
- This observation was made by Borgs et. al.
- If sufficient samples are generated, $\sigma(S)$ can be accurately estimated with high probability.
- To estimate $\sigma_T(S), \sigma_N(S)$, we randomly select a Target, Non-Target respectively.

EXPERIMENTS

Experiment Results

- Exact computation of $\sigma_T(S), \sigma_N(S)$ is #P-Hard

- Several techniques exists: Monte Carlo Simulations, Forward Influence Sketching, Reverse Influence Sketching(RIS)

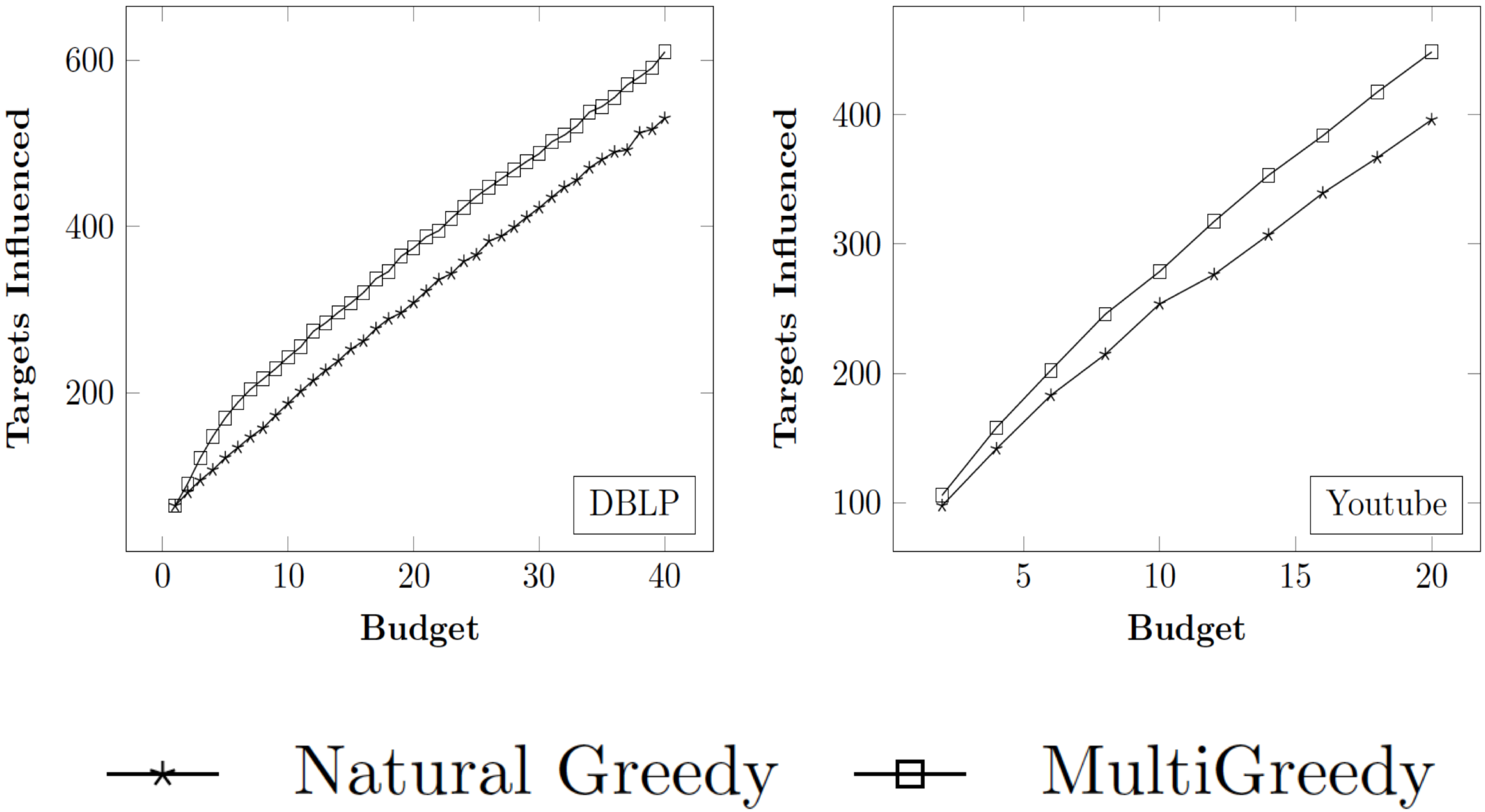
- We've used RIS based estimation.

- Our algorithms can be adapted to different methodologies of estimating the influence function

Datasets

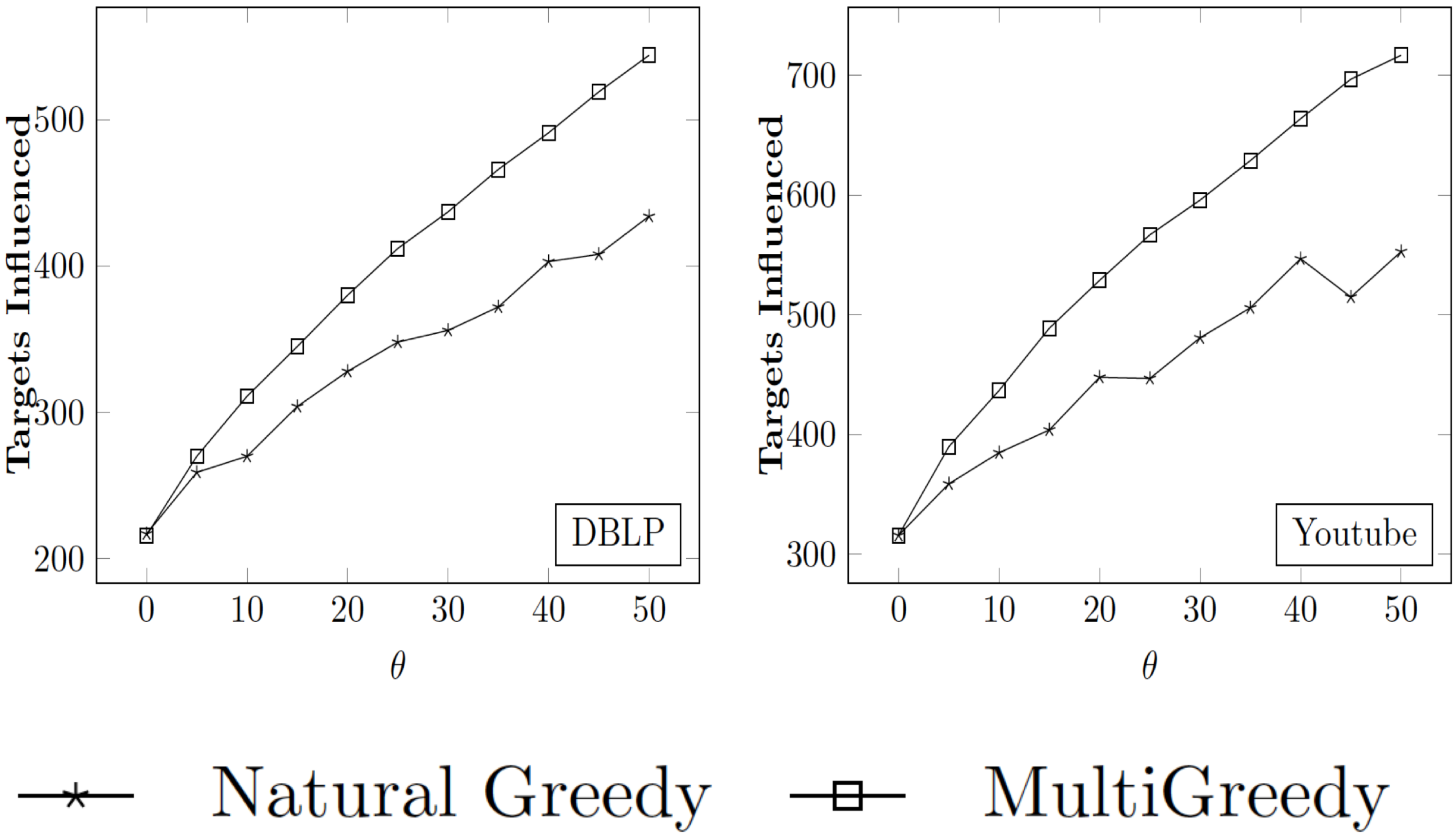
Network Name	**# Nodes**	**# Edges**
NetHept	15 k	62 k
Epinions	75 k	508 k
Amazon	334 k	925 k
DBLP	613 k	1.99 M
Youtube	1.13 M	2.98 M
Pokec	1.63 M	30.62 M

Budget Vs. Influence



$\theta = 10$

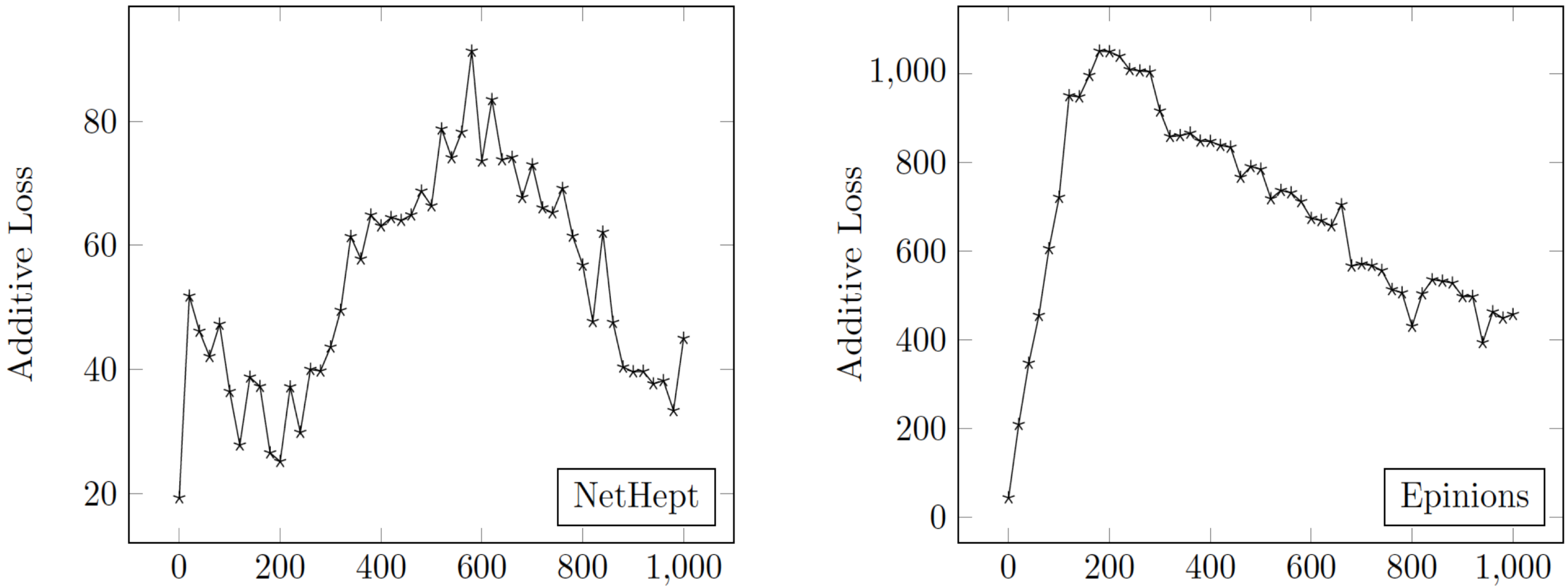
Threshold Vs. Influence



$k = 20$

Additive Loss in Natural Greedy

$$\sigma_T^{\theta}(S) \geq 0.63\ OPT - Additive\ Loss$$



$$\sigma_T^{\theta}(S) \geq 0.63\ OPT - Additive\ Loss$$

$$k = 20$$

Our Contributions

- Formulated the Constrained Influence Maximization (CIM) Problem
- Provided a theoretical analysis on hardness of CIM
- Studied the Greedy algorithm and proved its approximation guarantee involving an additive error
- Designed a novel MultiGreedy algorithm with an efficient implementation
- Experimentally evaluated Greedy, MultiGreedy algorithms on real world datasets

Future Work

- Can we design an algorithm that can tighten the additive error?
- Study how the additive error depends on the structure of the graph.

Thank you!

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